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MATHEMATICAL MODELING OF FLOWS INSIDE ROTATING BODIES MADE OF CELLULAR-POROUS MATERIALS

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A physicomathematical model is developed, which describes gas flows inside rapidly rotating bodies made of cellular-porous materials. Asymptotic and numerical solutions are obtained for some problems of forced centrifugal convection inside cylindrical cellular-porous bodies. The effect of the governing parameters (drag coefficient and dimensionless length of the cylinder) on characteristics and types of flows is considered.

Key words: *cellular-porous materials, vortex flows, mathematical modeling.*

Introduction. The development of manufacturing technologies of cellular-porous materials (CPM) offers vast prospects of their application in engineering for various burners and energy converters [1, 2]. The necessity of studying internal and external flows for bodies partly or completely made of these materials requires approaches to be developed to describe internal aerodynamics under various conditions. In particular, during rotation of CPM bodies, a forced convective flow caused by centrifugal forces is formed inside and around the bodies. An analysis of characteristics and modeling of such flows are interesting from the viewpoint of possible controlling mass and energy fluxes and forming desirable hydrodynamic structures.

Flows inside rotating porous materials were previously studied under the assumption that the angular velocity is low. This made it possible to obtain asymptotic and numerical solutions for some problems of thermal convection within the framework of the filtration theory and its modification with allowance for rotational motion [3–7]. Currently produced cellular-porous materials possess high permeability (about 0.95) and comparatively low drag. For practical applications indicated above, of interest is to consider processes with a high velocity of revolution of bodies (1000–5000 rpm). Under these conditions, assumptions of the linear filtration theory, which allow neglecting convective terms, are inapplicable. Therefore, the problem should be considered in a nonlinear formulation, i.e., with allowance for convective and inertial terms in equations of conservation of momentum.

The objectives of the present work are to develop a nonlinear physicomathematical model to describe gas flows inside rapidly rotating CPM bodies, to obtain some asymptotic solutions, and to analyze the influence of the governing parameters on characteristics of the flow.

Physicomathematical Model. We consider a CPM body rotating with a certain angular velocity. Because of centrifugal forces, a forced convective flow is formed inside this body with a corresponding flow formed outside the body. We have to determine the parameters of the internal gas flow, which is necessary, e.g., for finding loads acting on the body. An important problem is also determining the characteristics of the external air flow.

As the size of inhomogeneities in cellular-porous materials is much smaller than the characteristic microscopic size, the description can be made in terms of approaches of mechanics of heterogeneous media. The CPM structure is considered as a homogeneous permeable phase whose action on the second phase (liquid or gas) flowing through the first one is manifested only as a drag force. Because of high permeability, flow constriction inside the porous structure can be neglected. Then the interaction force reduces to friction drag only. We restrict our consideration

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to subsonic flows, the effects of compressibility, thermal convection, gravity force, and viscous dissipation being neglected at the first stage. Therefore, gas viscosity will be taken into account only in the term that describes the force of gas interaction with the porous structure.

First, we study a three-dimensional steady axisymmetric isothermal gas flow inside a porous cylindrical body rotating with a constant velocity. The equations in dimensionless variables in the laboratory (with respect to a motionless observer) cylindrical coordinate system (r, z, θ) fixed at the axis of revolution have the following form:

$$\frac{\partial ur}{\partial r} + \frac{\partial vr}{\partial z} = 0, \qquad \frac{\partial u^2 r}{\partial r} + \frac{\partial uvr}{\partial z} = -r\,\frac{\partial p}{\partial r} + w^2 - rf_r,$$
$$\frac{\partial uvr}{\partial r} + \frac{\partial v^2 r}{\partial z} = -r\,\frac{\partial p}{\partial z} - rf_z, \qquad \frac{\partial uwr}{\partial r} + \frac{\partial vwr}{\partial z} = -uw - rf_\theta. \tag{1}$$

Here p is the pressure, u, v, and w are the radial, longitudinal, and tangential components of velocity, and $\boldsymbol{f}$ is the specific volume force that describes gas interaction with the porous structure. The following characteristic scales of the problem are introduced: angular velocity of revolution Ω, characteristic length, which is the outer radius of the cylinder R, characteristic velocity $R\Omega$, gas density under standard conditions ρ_0, pressure scale $p_0 = \rho_0 R^2\Omega^2$, and time scale $1/\Omega$.

The particular form of the specific volume force depends on certain conditions. If the velocity of revolution is rather high, the force of interaction between the gas and the porous skeleton can be presented as a quadratic dependence $\boldsymbol{f} = K|\boldsymbol{v} - \boldsymbol{v}_s|(\boldsymbol{v} - \boldsymbol{v}_s)$. Here $\boldsymbol{v}_s = r\boldsymbol{i}_\theta$ is the velocity of a volume element of the rotating body and $K = kR$, where k is the drag coefficient depending on material properties, porosity, and gas viscosity. We will also consider a linear law, which is valid for low filtration rates, $\boldsymbol{f} = L(\boldsymbol{v} - \boldsymbol{v}_s)$, $L = \lambda/\Omega$, where λ is the coefficient in the filtration law. Beklemyshev [8] described the experimental results for the drag coefficient of various CPMs and a two-term dependence where the specific volume force $\boldsymbol{f}$ is presented as a linear combination of two formulas given above.

One-Dimensional Swirl Flows. Swirl flows whose parameters depend on the radius only are encountered, e.g., in disks that have an inner cavity and are closed by impermeable walls on butt-end faces [9]. When such a disk rotates, the gas freely flows from the outer area into the inner cavity of radius r_0, passes through the porous material, and leaves through the outer surface $r = 1$. For a steady flow, Eqs. (1) reduce to a system of ordinary differential equations

$$\frac{dur}{dr} = 0, \qquad \frac{du^2 r}{dr} = -r\,\frac{dp}{dr} + w^2 - rf_r, \qquad \frac{duwr}{dr} = -uw - rf_\theta, \tag{2}$$

which directly yield the quantity $u = q/r$ (q is a constant quantity characterizing the flow rate). The boundary-value problem for system (2) is posed in accordance with physical conditions. The pressure is determined with accuracy to a constant defined by conditions at infinity. If a constant flow rate of the gas through the porous body is somehow maintained, we have $u = q_0/r$. In this case, one condition for w is sufficient, e.g., $w(r = r_0) = 0$ (absence of swirling of the incoming flow on the inner surface). If the body is located in free space, an additional condition determining the flow rate can be obtained from the Bernoulli integral for the incoming and outgoing flows. (In [9], the corresponding condition is written in the general form with allowance for friction and pressure losses.)

For a linear law of drag [$f_r = Lu$ and $f_\theta = L(w - r)$], the solution is determined analytically; for $w(r_0) = 0$, it has the form

$$w = \Big(r - \frac{1}{\alpha r}\Big) - \frac{r_0}{r}\Big(r_0 - \frac{1}{\alpha r_0}\Big)\exp\left[-\alpha(r^2 - r_0^2)\right], \qquad \alpha = \frac{L}{2q}. \tag{3}$$

For a quadratic law of drag, we have $f_r = Ku\sqrt{u^2 + (w - r)^2}$ and $f_\theta = K(w - r)\sqrt{u^2 + (w - r)^2}$, and the problem reduces to solving the equations

$$u = \frac{q}{r}, \qquad \frac{dW}{dr} - \frac{2r}{q} = -KW\sqrt{1 + W^2}, \qquad W = -\frac{r(w - r)}{q}. \tag{4}$$

For small r, W is also small; then, with accuracy to W^2, we can replace the right side of the second equation in (4) by $-KW$ and obtain the asymptotic solution

$$w = r - \frac{2}{K} + \frac{2}{K^2 r} - \frac{r_0}{r}\Big(r_0 - \frac{2}{K} + \frac{2}{K^2 r_0}\Big)\exp\left[-K(r - r_0)\right]. \tag{5}$$

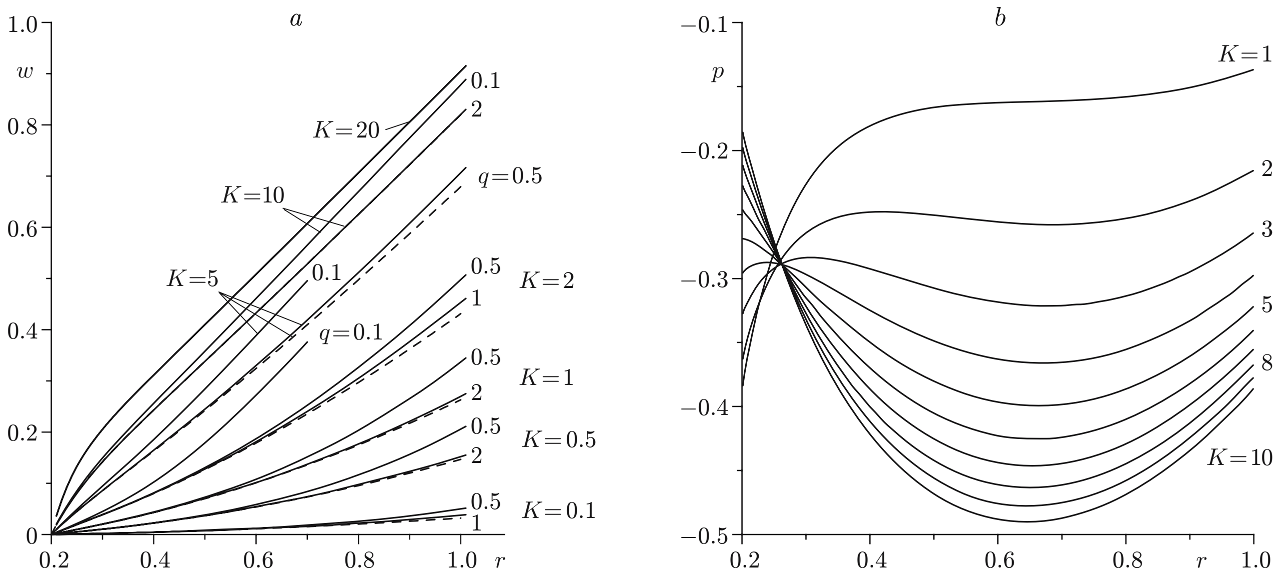

Fig. 1. Azimuthal velocity (a) and pressure (b) of a plane swirl flow inside a disk with closed end faces ($r_0 = 0.2$).

The distributions of the azimuthal velocity obtained by numerically solving Eqs. (4) with different values of q are plotted by solid curves in Fig. 1; the dashed curves refer to the asymptotic solutions (5). For $q > 1$, good agreement is obtained in the entire domain under consideration, up to the outer boundary. As the drag parameter K increases, the range of admissible values of q becomes wider. In particular, for $K = 20$, the asymptotic solution and the curves obtained numerically for $q = 0.1$ and $q = 10$ almost coincide. The functions $w(r)$ in the dependences on L for the linear law of drag behave in a similar manner.

If there are no obstacles (e.g., casing) restricting gas motion around the rotating porous body, the flow rate can be determined with the use of the Bernoulli integral for the internal flow:

$$\Delta p = p_1 - p_0 = (u_0^2 + w_0^2 - u_1^2 - w_1^2)/2. \tag{6}$$

It follow from (6) and the second equation of system (2) that

$$-w_1^2 = 2 \int\limits_{r_0}^{1} \Big(\frac{w^2}{r} - f_r\Big) dr. \tag{7}$$

It is possible to solve Eq. (7) together with system (2) for both the linear and the quadratic law of drag numerically, by the method of iterations. The calculated dependences of the flow rate on K for different values of r_0 are shown in Fig. 2a by the solid curves.

To obtain an approximate value of the flow rate without solving the full problem, one can use the following asymptotic estimates. For small $\alpha = L/(2q)$, Eqs. (3) and (7) predict $q = \sqrt[3]{3L^2/(32 \ln(1/r_0))}$. This approximation is admissible for $\alpha < 0.1$ and imposes a stringent restriction on the value of L (e.g., up to 0.003 for $r_0 = 0.05$ and up to 0.0006 for $r_0 = 0.2$). In the opposite case with $\alpha \gg 1$, with allowance for the fact that the solution $w(r)$ is rather close to a linear function for $\alpha = 1/(2r_0^2)$ and rather small r_0, the estimate $q \approx 1/(L \ln(1/r_0))$ is valid, which ensures a good approximation for $r_0 < 0.2$ and $L > 8$. For the quadratic law of drag, the asymptotic solution (5) for $(w - r)/q \ll 1/r$ yields an approximate estimate

$$q = \Big\{\frac{r_0}{2K(1 - r_0)}\Big[w_1^2 + 2 \int\limits_{r_0}^{1} \frac{w^2}{r}\, dr\Big]\Big\}^{1/2}. \tag{8}$$

Solution (8) (dashed curves in Fig. 2a) can be used to determine the flow rate for high values of K and rather small values of r_0: $r_0 < 0.5$ and $K > 5$. For both laws of drag, the flow-rate dependence on the drag parameter $K(L)$

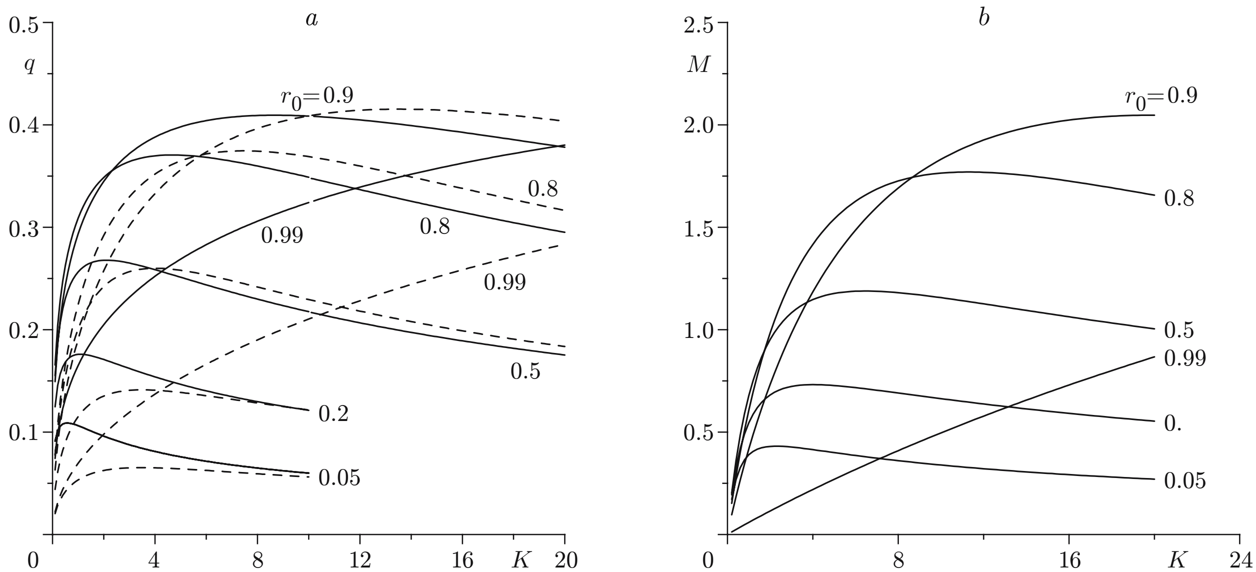

Fig. 2. Flow rate (a) and moment (b) versus the drag parameter and inner radius.

is nonmonotonic and has a maximum point (optimal structure) whose position and magnitude depend on r_0. The second specific feature implies that an increase in the inner radius to 0.8–0.9 increases the flow rate and shifts the maximum point toward higher values of L and K and higher values of the inner radius (about 0.9). These features of the flow-rate behavior were also noted in [9].

The moment of the force necessary to sustain rotation of the porous disk with a given constant angular velocity is compensated by the integral moment of the azimuthal drag force and the external drag force, which is substantially smaller. Neglecting the latter, we obtain

$$M = \int_{r_0}^{1} 2\pi r^2 f_\theta \, dr.$$

The moment as a function of the parameter K is plotted in Fig. 2b. This dependence has a maximum point too, but it does not coincide with the maximum point for the flow rate. The maximum value of the flow rate is located on the ascending part of the curves in the dependences $M(K)$ [and also $M(L)$]; hence, the quantity q/M also has a maximum point in terms of L or K for each fixed value of r_0. The maximum point indicates the optimal structure in terms of the flow rate to moment ratio.

The pressure p is determined from Eq. (1) and, for $p_\infty = 0$, can be presented as

$$p = -\frac{q^2}{2r^2} + \int_{r_0}^{r} \Big(\frac{w^2}{r} - f_r\Big) \, dr.$$

In Fig. 1b, the distributions of p are plotted for the case where the rotating disk is located in free unbounded space, and the flow rate is determined with allowance for Eq. (7). The pressure profiles depend qualitatively both on the drag parameters (which is seen in Fig. 1b) and on the inner radius. For $r_0 = 0.2$ and low values of K (or L), the pressure monotonically increases and reaches a maximum at the exit. With increasing drag parameter, local maximums and local minimums inside the domain appear. The point of the local maximum is shifted to the left with increasing L and K or with increasing r_0 and reaches the boundary $r = r_0$. The point of the local minimum is shifted to the right with increasing r_0 and reaches the boundary; hence, for $r_0 = 0.8$, the pressure decreases with increasing r for all $K > 1$. As it follows from the analysis of the second equation of system (2), the complicated behavior of pressure is caused by the simultaneous influence of the convective term $du^2r/dr = -(q/r)^2$, centrifugal force depending on w^2, and drag force f_r. The flow rate q is also a complex function of the drag parameter and r_0. It is possible to determine conditions for the pressure inside the disk to be close to a constant value. For instance,

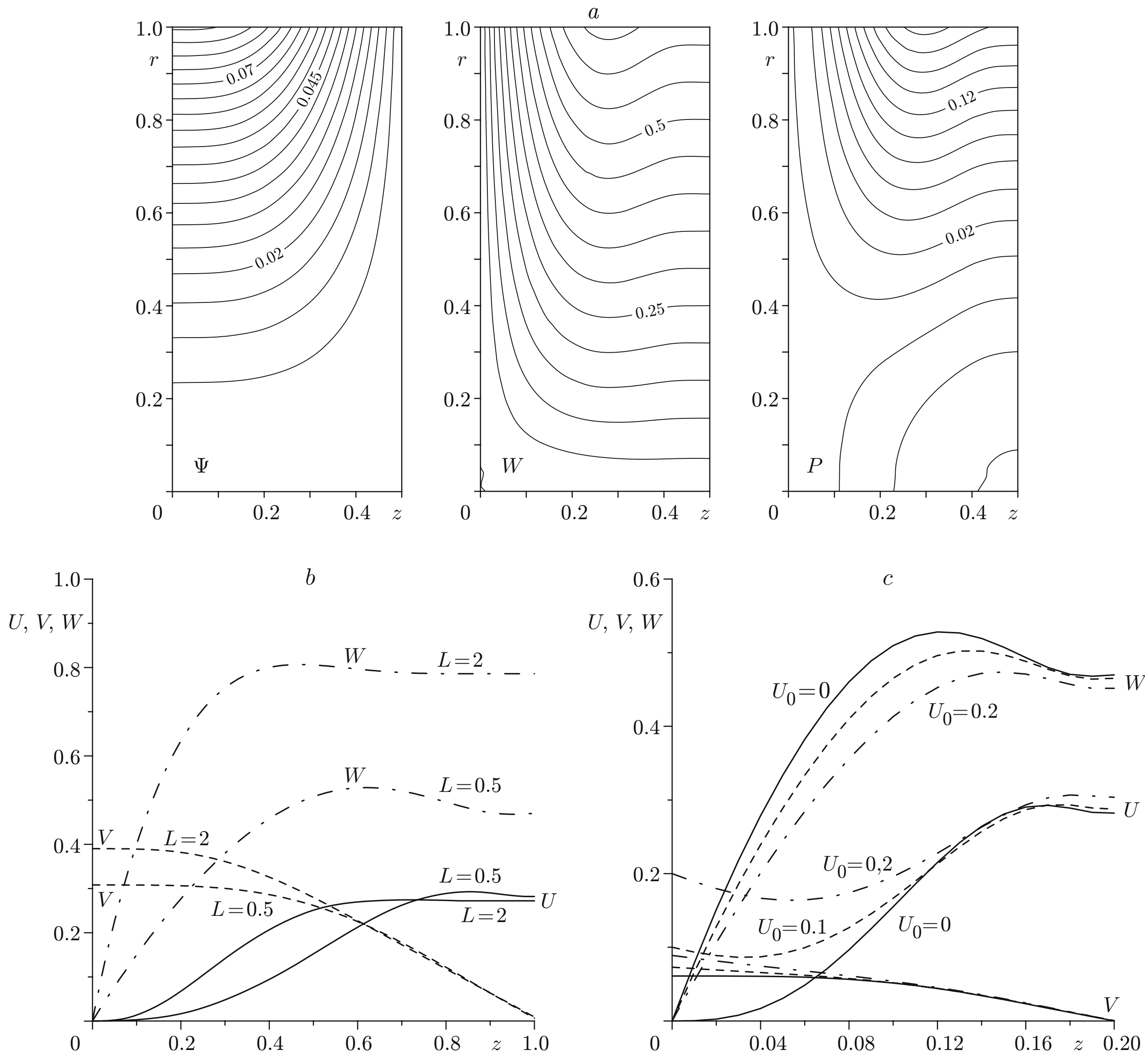

Fig. 3. Flow inside a rotating cylindrical body made of a CPM with a linear law of drag: isolines of parameters in the z–r plane (a) and dependence of solutions on the drag parameter L (b) and on inclination of the incoming flow U_0 (c).

the smallest difference between the maximum and minimum values is observed for $K \approx 3.5$ ($r_0 = 0.2$), $K \approx 1$ ($r_0 = 0.5$), and $K \approx 0.4$ ($r_0 = 0.8$). The total pressure $P = p + (u^2 + v^2 + w^2)/2$ under given boundary conditions equals zero for $r = r_0$ and $r = 1$; therefore, there is always a local minimum inside the domain. The higher the drag parameter, the lower the value of P at this point.

Spatial Swirl Flows with Axial Symmetry. Let us further consider rotation of a porous cylinder of finite length H, whose surface is completely permeable. Under the action of centrifugal convection forces, the inflow occurs through the butt-end planes $z = 0$ and $z = 2z_0$ ($z_0 = H/2R$), and the outflow proceeds through the surface $r = 1$. To obtain complete information about the gas flow through porous bodies, generally speaking, one has to solve a conjugate problem, i.e., consider the internal and external flows in a coupled manner. Under certain conditions, particular solutions can be obtained for the internal flow. These solutions can be further used for testing the numerical method for calculating the two-dimensional internal flow in the general case and for comparisons with the solution of the conjugate problem for internal and external flows.

For the linear law of drag, the equations of steady motion admit a particular solution of the form $u(r,z) = rU(z)$, $v(r,z) = V(z)$, $w(r,z) = rW(z)$, and $p(r,z) = P(z) + P_0 r^2$, and the functions $U(z)$, $V(z)$, $W(z)$, and $P(z)$ satisfy the system of ordinary differential equations

$$\frac{dV}{dz} = -2U, \qquad \frac{dU}{dz} = \frac{W^2 - U^2 - LU - 2P_0}{V},$$
$$\frac{dP}{dz} = 2UV - LV, \qquad \frac{dW}{dz} = -\frac{2UW + L(W-1)}{V}. \tag{9}$$

The incoming flow ($z = 0$) has three velocity components. The radial component is determined by the prescribed constant U_0; $u(r,0) = U_0 r$ and $P_0 = -0.5U_0^2$. The value of the axial component of velocity at the entrance is unknown, but it should be equal to zero at the axis of symmetry ($z = z_0$). It is assumed that the azimuthal velocity at the entrance is equal to zero (no swirl of the external flow), and the pressure is determined from the Bernoulli integral. The corresponding boundary conditions for system (9) are

$$z = 0\colon \quad U = U_0, \quad W = 0, \quad P = p_\infty - 0.5V^2; \qquad z = z_0\colon \quad V = 0. \tag{10}$$

The solution of the boundary-value problem (9), (10) is found by the numerical shooting method: a value of $V(0)$ is chosen for which the solution of the Cauchy problem satisfies the condition $V(z_0) = 0$.

The calculated results are plotted in Fig. 3. Figure 3a shows the isolines of the main parameters for $L = 1$, $U_0 = 0$, and $z_0 = 0.5$; the dependences of the sought functions on the parameter L for $U_0 = 0$ and $z_0 = 1$ are displayed in Fig. 3b. It is seen that the azimuthal velocity is the most sensitive characteristic in terms of the drag coefficient. The nonmonotonic behavior of the velocity components u and w (correspondingly, dynamic pressure and total pressure P) at the exit (side) surface of the cylinder should be noted, which was also observed in experiments.

Note that the condition of parallelism of the incoming flow and the axis of symmetry $U_0 = 0$ seems to be limited to a certain extent, because it does not ensure conjugation with the external flow. Therefore, we considered the effect of flow deflection (value of U_0) on the solution; the results are plotted in Fig. 3c ($L = 0.5$ and $z_0 = 0.2$). The axial velocity changes insignificantly, and the radial velocity becomes nonmonotonic with increasing U_0.

The influence of the geometrical parameters (dimensionless length of the cylinder) can be estimated by comparing the curves corresponding to $U_0 = 0$ and $L = 0.5$ in Figs. 3b and 3c. A decrease in the dimensionless length of the cylinder z_0 from 1 to 0.2 affects the value of the axial velocity V and has almost no influence on the distributions and maximum values of U and W.

The solutions obtained and their properties described above can be used to solve the full problem of internal and external flows and as test solutions in developing a numerical algorithm for calculating the problem with a general law of drag.

Conclusions. The study performed was crowned by the following results.

A physicomathematical model was developed to study incompressible gas flows inside rapidly rotating bodies made of cellular-porous materials.

Asymptotic and numerical solutions were obtained for one-dimensional swirl flows inside cellular-porous materials with different laws of drag; a qualitatively different behavior of pressure versus the drag parameter was demonstrated.

Numerical solutions were obtained for a two-dimensional flow inside an open rotating cylindrical body with a linear law of drag of its structure; the influence of the geometrical parameters of the problem and the drag parameter on the solution was considered; the solutions were found to depend on the incoming flow direction, which indicates that it is necessary to solve a conjugate problem with allowance for the external flow.

The model developed and the solutions obtained can be used to solve the full problem of internal and external gas flows in the case of rotation of CPM bodies. The model can also be extended with allowance for viscous dissipation, heat transfer, and chemical reactions.

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